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Transmission Characteristics of a Three-Conductor Coaxial Transmission Line with Transpositions

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This paper describes in detail the manufacture of, measurements on, and computations about a three-conductor coaxial cable with transpositions. By reducing skin-effect losses in the central conductor, this three-conductor line achieves 17 per cent lower attenuation at a particular frequency than does a two-conductor line of the same outer diameter. The matrix method used for analysis can easily be extended to n*-conductor lines. Suggestions for making improved three-conductor coaxial lines are made.*

I. SUMMARY

Most of the loss in a simple coaxial cable is due to series resistance in the center conductor. At high frequencies this resistance is aggravated by the skin effect, which causes conduction currents to concentrate in a thin layer on the surface of the conductor. Because of the skin effect, the effective cross-sectional area of a conductor is smaller than its geometric cross section at high frequencies.

The effective cross section of the center conductor of a new coaxial line has been increased by dividing the conductor into two parts, a solid central member and a concentric thin shell insulated from the center member. Over a certain range of frequencies, conduction currents are distributed throughout the whole thickness of the thin cylindrical shell

and a layer one skin-depth thick on the surface of the solid center member.

The total current in the composite center conductor is divided into two components, one flowing in the solid center member and one flowing in the thin cylindrical shell. The effective resistance of the composite center conductor is less than that of a solid conductor only when these two current components are approximately equal both in phase and in magnitude, and then only over a limited frequency range. The desired division of current has been achieved by electrical transposition of the two parts of the inner conductor at intervals of less than one-eighth of a wavelength.

Several thousand feet of such a three-conductor transmission line have been constructed with transpositions at various intervals. Detailed measurements and calculations have been made, and the three-conductor cable has been compared directly with a similarly constructed conventional two-conductor cable. The three-conductor cable has lower attenuation than the two-conductor cable at frequencies between 1 and 10 mc with transpositions 9 feet apart or less. Near 4 mc, the three-conductor cable has 17 per cent less attenuation than does the two-conductor cable. The agreement between measurements and computations is excellent. The improvement in the three-conductor cable can be increased to 25 per cent by changing the relative dimensions of the parts of the center conductor, and the frequency range over which it surpasses a two-conductor cable can be increased thereby to 20:1.

II. BACKGROUND

2.1 *Introduction*

A transmission line with three parallel conductors can support two modes of transmission at low frequency.[1] Each mode has a well-defined phase velocity and attenuation at every frequency. It might be assumed that the transmission losses on such a line are minimized if power is transmitted only in the mode having the lower attenuation. If the line is sufficiently long, this is true; but for short sections of line it is not. The losses in a short section of line may be reduced, under some circumstances, by a judicious combination of pure modes.

The reason why the losses may be smaller for a combination of pure modes than for any of the modes separately depends on constructive and destructive interference. If two pure modes propagate with different wave velocities, the currents which they induce in the conductors will

be successively in phase, out of phase, in phase again, and so on. When the currents are in phase, the losses are greater than when the currents are out of phase. Over a long length of line, the phases will pass through many cycles, and the amount of power lost will average out to be the same as if the pure modes were propagated independently.

If two modes are propagated over a short length of line so that there is substantial cancellation of currents, the losses will be reduced *so long as the currents do cancel substantially*. When the differential phase shift between modes begins to be appreciable, it will be necessary to terminate the line, draw off the power and launch it in a new length of line. If this is not done, the reduction in losses will be offset precisely by an increase in losses when the currents of the two modes become in phase.

This phenomenon can be used to combat losses due to skin effect in a coaxial line. In a simple coaxial line, most of the effective resistance of the line is in the center conductor. This effective resistance can be reduced if the center conductor is replaced by a smaller center conductor surrounded by a thin coaxial tubular conductor, provided the magnitudes and phases of the currents in these two inner conductors bear the right relation. As will be shown, this can be accomplished by transposing the two parts of the center conductor, which is equivalent to the process described in the previous paragraph.

The problem of losses in a three-conductor transmission line has much in common with that of losses in a two-conductor line that has standing waves. The three-conductor line has four waves, one forward and one backward in each distinct mode of propagation, and these four waves may beat in various forms of destructive and constructive interference. In order to get an intuitive grasp of the performance of such a line, it is convenient to subdivide the problem into two parts: finding the loss in the line for a given distribution of currents, and determining how the current distribution which yields the lowest losses for a given power transfer can be attained. The answer to the first question tells how much a coaxial line can be improved; the answer to the second tells how to do it.

2.2 *Losses in a Three-Conductor Coaxial Having a Given Current Distribution*

In the next section an accurate calculation of properties of a three-conductor coaxial will be made. But a good idea of how much the loss may be reduced in this kind of line, under the proper conditions, may be obtained by assuming a simple model for the line, as shown in Fig. 1, and also assuming that the currents in the three conductors are known.

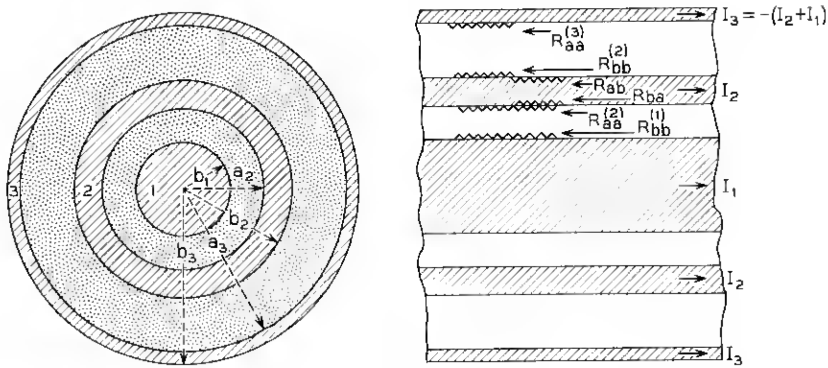


Fig. 1 — Three-conductor coaxial transmission line: cross section and schematic longitudinal section showing surface resistance.

The loss in such a line is given approximately by

$$L = \tfrac{1}{2} \mid I_1 \mid^2 R_{bb}^{(1)} + \tfrac{1}{2}[\mid I_1 \mid^2 R_{aa}^{(2)} + (I_1\bar{I}_3 + \bar{I}_1 I_3) R_{ab}^{(2)} + \mid I_3 \mid^2 R_{bb}^{(2)}] + \tfrac{1}{2} \mid I_3 \mid^2 R_{aa}^{(3)},$$

where I_1 and I_3 are the current amplitudes in the inner and outer conductors respectively and the resistances are the surface resistances calculated from formulas (83) in Schelkunoff's paper[2] on cylindrical transmission lines.* The superscripts in parentheses refer to the particular conductor being considered; R_{aa} is the resistance of the inner surface of a tube per unit length to current which is returned entirely inside the surface, R_{bb} is the resistance of an outer surface of a tube per unit length to current which is returned entirely outside the surface and R_{ab} is the transfer resistance from one surface of a tube to the other.

Of the five terms in the equation above, the first is the ohmic loss in the center conductor. The second, third and fourth terms are the loss in the intermediate conductor. The resistances R_{aa} , R_{ab} and R_{bb} are real functions.

It is obvious that ouly one of the five loss terms depends on the relative phases of I_1 and I_3 . This is the third term, which contains the factor

$$(I_1\bar{I}_3 + \bar{I}_1 I_3) = 2 \mid I_1 \mid \mid I_3 \mid \cos \theta,$$

* The reader should note that the first-order correction terms in Schelkunoff's formulas (82) are in error; formulas (75), from which they were derived, are correct. Briefly, Z_{aa} , Z_{ab} , Z_{ba} and Z_{bb} are the ratios of longitudinal electric field strength on the inner or the outer surface of a tubular conductor to the part of the total current returning either inside or outside the tube. The real parts are denoted by R with the same subscripts. For details, see Ref. 2, pp. 554–558.

where θ is the phase difference between I_1 and I_3. Any reduction in losses depends on having R_{ab} large and choosing I_1 and I_3 so that the term containing R_{ab} is negative.

These theoretical calculations show that the losses in a coaxial transmission line can be reduced by using a third conductor. Suppose, for example, that the skin depth, the thickness of the inner dielectric layer and the thickness of the intermediate conductor, are all small relative to the radius of the center conductor. Then the various loss formulas are simplified and it is easy to compare the losses in the two innermost conductors with the loss in a solid conductor of the same over-all dimensions. This comparison is made in Fig. 2. Here, the dotted and dashed curves show the resistances of the two conductors compared to the resistance of a single solid conductor. For the dotted curve, the distribution of currents is chosen to minimize the resistance. The required distribution is shown in the solid curve, which gives the best ratio I_3/I_1. The dashed curve shows the relative resistance if the current is divided equally in the two inner conductors. In either case, the resistance may be reduced to 65 per cent of that of a solid wire for a particular frequency.

In a normal coaxial line 78 per cent of the effective resistance is in the

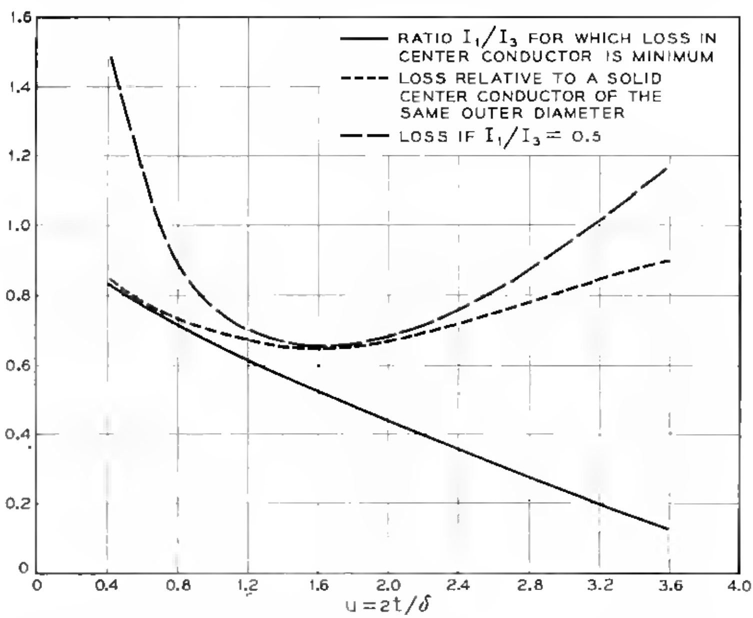


Fig. 2 — Comparison of losses in simple and compound center conductors for certain current distributions.

center conductor and 22 per cent is in the outer conductor. When the center conductor resistance is reduced 35 per cent, the attenuation is reduced not 35 per cent but roughly 78 per cent $\times$ 35 per cent = 27 per cent.

From the curves of Fig. 2 we can conclude that, if the current in a suitably proportioned three-conductor coaxial line can be distributed so that the two innermost conductors carry equal currents, at one particular frequency the attenuation can be reduced to 73 per cent of the attenuation of a two-conductor coaxial line of the same over-all dimensions and made of the same materials. The attenuation of the three-conductor line will be less than that of the two-conductor line over a 20:1 range of frequencies, but at extremely low or extremely high frequencies it will have higher attenuation.

2.3 *How Can the Desired Current Distribution Be Achieved?*

The previous section shows that a coaxial transmission line having three conductors, the two innermost of which are very close together, will have lower losses than a conventional coaxial cable if the currents in the two innermost conductors are approximately equal. Two independent ways to achieve this equal division of current have been suggested. One way* is to alter the dimensions and dielectric constants so that the line has a natural mode of propagation with approximately the desired current distribution. The other proposal is to transpose conductors in the manner of a litz wire to get the desired distribution by brute force, so to speak. This paper describes computations and experiments with a line having such transpositions.

2.4 *History*

The idea of transposition between a solid cylindrical conductor and a concentric tube is at least as old as U. S. Patent 1,088,902, issued on March 3, 1914, to P. V. Hunter. Fig. 4 of that patent shows a transposition similar to Fig. 15 of the present text, and the exact wording of the specification states:

"*If it is desired to arrange the feeders so that the leads have equal impedance, the cable may be divided into an even number of equal lengths, the inner conductor* a *of one length being, as indicated in Fig. 4* [of the patent] *connected to the other conductor* b *of the adjacent length. This arrangement also prevents alteration to the division of current between the two leads by unequal inductive effects on the two leads of currents in adjacent feeders or earth.*"

* Proposed by H. S. Black and S. P. Morgan, Jr.

Hunter was concerned with power distribution systems. The increase in resistance at high frequency resulting from the skin effect is not mentioned in his patent. This is reasonable, for at a frequency of 50 cycles per second the increase in resistance of a copper conductor even as large as one-half inch in diameter is only 0.34 per cent. One can easily understand, therefore, why Hunter did not apply his technique to the reduction of skin resistance.

Later, when the nonuniformity of current distribution at high frequencies became a matter of concern, stranded conductors[3,4,5] were used rather than transposed coaxial cylinders. At frequencies below one megacycle per second, stranded conductors may be quite advantageous. Above that frequency, the fineness and large number of the strands required makes the technique less rewarding.

The work reported in this paper stems from a systematic search for ways to combat skin and proximity effects. The beginning of this search was stimulated largely by W. H. Doherty during his tenure as Director of Research–Electrical Communications at Bell Telephone Laboratories.

2.5 *Early Experimental Results*

The first three-conductor coaxial line with transpositions was made with a center conductor devised by W. McMahon and fabricated by him and G. R. Johnson as shown in Fig. 3. Starting with a bare wire 30 mils in diameter, flat sections 8 in. long, 120 mils wide and 5 mils thick were formed at regular intervals along the wire so that they were separated by a length slightly greater than 8 in. This was done by placing the wire on a hardened steel plate and passing the plate and wire through a rolling mill. The flat sections were then formed into channels in a simple die. Then the round sections of the wire were insulated with a wrapping of plastic tape. Finally, two such conductors were placed side by side so that the flat sections (now channels) of one were matched with the round sections of the other, as shown in the figure. The round sections were laid in the channels, and the whole structure was passed through a die of diameter such that it caused the flat sections to be wrapped completely around the insulated round sections. Thus, the conductors alternate as inner and outer conductors along the line, as shown in the figure.

Resistances measured at various frequencies were compared with computations based on the approximate formulas of Section 2.2, which were, at that time, the best available. The results are shown in Figs. 4, 5 and 6. While the correspondence between the measured and computed

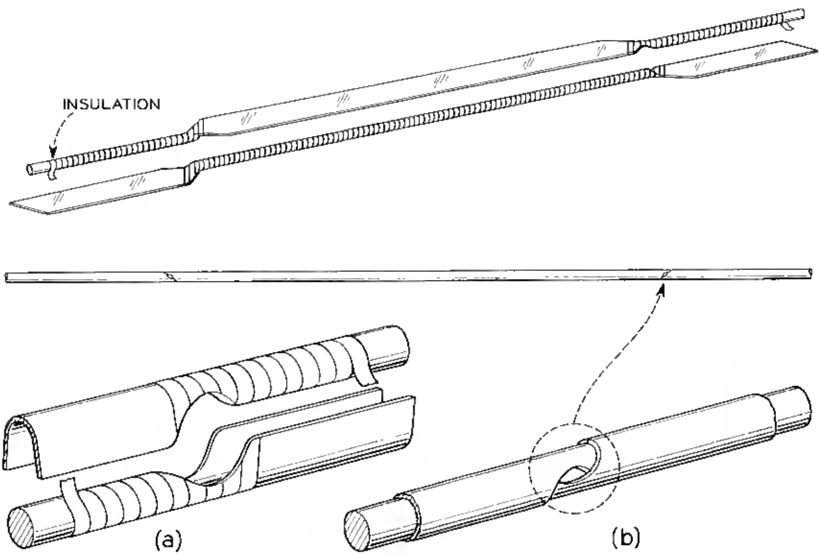


Fig. 3 — Method suggested by W. McMahon for making a transposed compound conductor.

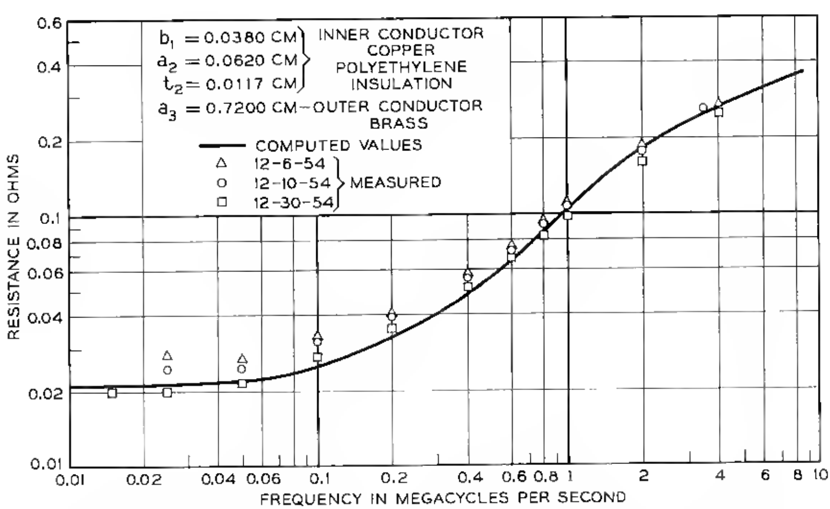


Fig. 4 — Effective resistance of a 1-meter length of coaxial line with a transposed copper center conductor.

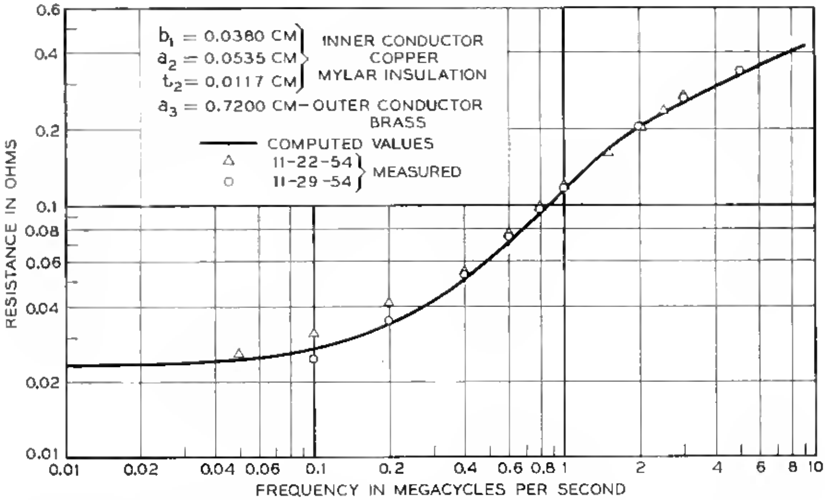


Fig. 5 — Effective resistance of a 1-meter length of coaxial line with a transposed copper center conductor of different size.

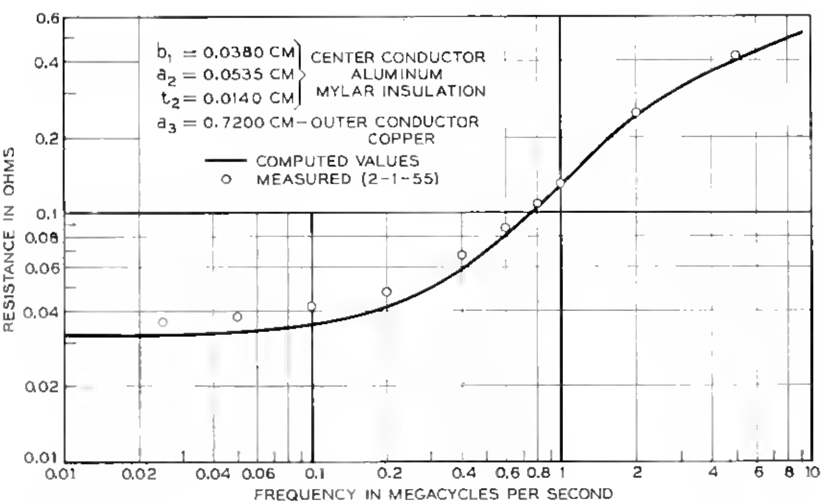


Fig. 6 — Effective resistance of a 1-meter length of coaxial line with a transposed aluminum center conductor.

values is not perfect, it is clear that the computed curves are a good qualitative approximation to the measured values.

At that time, it was decided to make a long test sample for further measurement. The experimental length was chosen so that: (1) different transposition intervals could be studied, (2) attenuation could be measured directly in a straight-through transmission measurement, (3) termination effects would be small in comparison to transmission effects and (4) a control experiment on a two-conductor coaxial line could be easily carried out. As this cable was being fabricated and measured, a detailed analysis and computation procedure was worked out. Actually, the measurements were completed six months before the computations. The results of this last experiment and computation are the main subject of this paper.

2.6 *Plan of the Paper*

Following this introduction, we shall make a thorough analysis of the three-conductor transposed line. In Section III, the propagation properties of a uniform three-conductor line are expressed in matrix form. In Section IV, a unit of transposed line, made by combining two equal lengths of this uniform line and a transposition, is described. A cascade of these units is then analyzed as a periodic transmission medium to find its new traveling waves and propagation constants. In general, the expressions are complicated and numerical analysis is necessary to obtain useful results. However, if the spacing between transpositions becomes very small, quite tractable forms appear. Further, an approximation to these forms yields a very simple understandable picture of the behavior of these lines. Following this, Section V is a description of experimental work and final results. Details of calculation and tables of results are given in Section VI.

III. PROPAGATION IN UNIFORM THREE-CONDUCTOR COAXIAL LINES

Consider the line with transpositions to be made up of a large number of equivalent sections each containing a certain length $2l$ of transmission line and one transposition. For the sake of preserving symmetry, it is convenient to place the transposition in the center, flanked by two equal lengths l of line, as in Fig. 7.

It is convenient to use matrix notation to describe the transformation of voltages and currents from one end of each of these elements to the other because the matrix of the transformation of any number of linear networks in cascade is simply the product of the matrices of the individ-

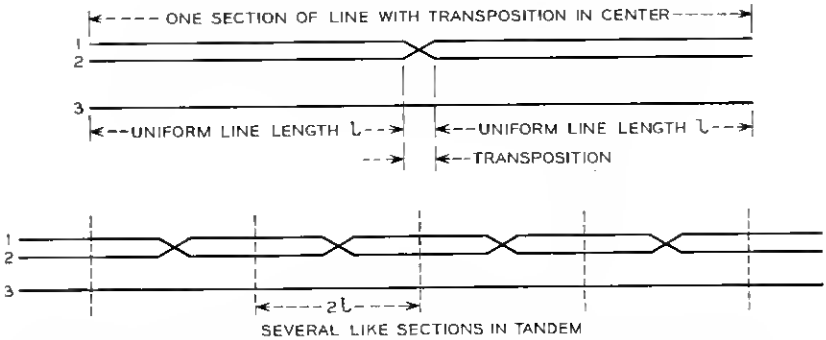


Fig. 7 — Schematic representation of a three-conductor transmission line with several transpositions.

ual transformations, even though the individual networks are not identical (Ref. 6, p. 337).

First, we will obtain a suitable matrix for the length l of uniform line, starting from the differential equations. Carson and Gilbert[1] and Schelkunoff[2] have shown how to derive a first integral of Maxwell's equations for a structure consisting of uniform concentric cylinders and shells. For the purpose of studying three-conductor coaxial lines it is only necessary to adapt Schelkunoff's general results to the specific case at hand.

Suppose the cross section of the transmission line is as shown in Fig. 1. Let the conductors be numbered, from the innermost out, 1, 2 and 3. Let the currents in the several conductors be $i_1e^{j\omega t}$, $i_2e^{j\omega t}$ and $i_3e^{j\omega t}$. Note that small letters are used instead of the capital letters used before, and that the dependence on longitudinal distance x is included. Let the voltages of the various conductors be $v_1e^{j\omega t}$, $v_2e^{j\omega t}$ and $v_3e^{j\omega t}$. To avoid ambiguity, we may suppose that voltages are measured radially inward from the outermost conductor. It follows that $i_3 = -i_1 - i_2$ and $v_3 = 0$, and we can eliminate i_3 and v_3 when desirable.

Following Schelkunoff, in analogy to his equation (52), we can find

$$\begin{aligned} \frac{j\omega\mu i_1}{2\pi}\log\frac{a_2}{b_1} &= E_x(a_2) - E_x(b_1) - \frac{\partial}{\partial x}(v_2 - v_1), \\ \frac{j\omega\mu(i_1 + i_2)}{2\pi}\log\frac{a_3}{b_2} &= E_x(a_3) - E_x(b_2) - \frac{\partial}{\partial x}(v_3 - v_2), \end{aligned} \tag{1}$$

$$\begin{aligned} v_2 - v_1 &= -\frac{1}{2\pi(g + j\omega\epsilon)}\log\frac{a_2}{b_1}\,\frac{\partial i_1}{\partial x}, \\ v_3 - v_2 &= -\frac{1}{2\pi(g + j\omega\epsilon)}\log\frac{a_3}{b_2}\,\frac{\partial}{\partial x}(i_1 + i_2). \end{aligned} \tag{2}$$

Here a_n and b_n are the inside and outside radii, respectively, of conducting shell n, and $E_x(a_n)$, $E_x(b_n)$ are the longitudinal electric field strengths at the inner and outer surfaces of the nth conductor.

To these equations must be added Schelkunoff's equation (74) (modified appropriately in notation) for determining the surface electric fields E_x:

$$\begin{aligned} E_x(a_n) &= Z_{aa}{}^{(n)} I_a{}^{(n)} + Z_{ab}{}^{(n)} I_b{}^{(n)}, \\ E_x(b_n) &= Z_{ba}{}^{(n)} I_a{}^{(n)} + Z_{bb}{}^{(n)} I_b{}^{(n)}, \end{aligned} \tag{3}$$

where $I_a{}^{(n)}$ and $I_b{}^{(n)}$ are the parts of i_n which flow back inside and outside the nth conductor and the Z's are the artificial surface impedances,* the resistances of which were used above. We see from Fig. 1 that

$$\begin{aligned} I_a{}^{(n)} + I_b{}^{(n)} &= i_n, \\ I_a{}^{(n)} &= -\sum_1^{n-1} i_n, \\ I_b{}^{(n)} &= -\sum_{n+1}^{3} i_n = \sum_1^n i_n = -I_a{}^{(n+1)}, \\ I_a{}^{(1)} &= 0. \end{aligned} \tag{4}$$

The terms on the left of the first two equations (1) above are the inductive reactance voltages resulting from the magnetic field between conductors. To simplify the writing, let X_{a1} and X_{a2} be these reactances so that

$$\begin{aligned} \frac{\omega\mu}{2\pi} \log \frac{a_2}{b_1} &= X_{a1}, \\ \frac{\omega\mu}{2\pi} \log \frac{a_3}{b_2} &= X_{a2}. \end{aligned} \tag{5}$$

In the second pair of equations (2), voltages and currents are related by the radial admittances Y_1 and Y_2 of the dielectric between conductors. That is,

$$\begin{aligned} \frac{2\pi(g + j\omega\epsilon)}{\log(a_2/b_1)} &= Y_1, \\ \frac{2\pi(g + j\omega\epsilon)}{\log(a_3/b_2)} &= Y_2. \end{aligned} \tag{6}$$

* Formulae for the surface impedances for the cylindrical line are given in Section VI.

Equations (1) and (2) may now be written

$$
\begin{aligned}
\frac{\partial i_1}{\partial x} &= \qquad\qquad\qquad\qquad\qquad\qquad Y_1 v_1 \qquad - Y_1 v_2, \\
\frac{\partial i_2}{\partial x} &= \qquad\qquad\qquad\qquad\qquad - Y_1 v_1 + (Y_1 + Y_2) v_2, \\
\frac{\partial v_1}{\partial x} &= (Z_{22} + Z_{11} - 2Z_{ab}) i_1 + (Z_{22} - Z_{ab}) i_2, \\
\frac{\partial v_2}{\partial x} &= \qquad (Z_{22} - Z_{ab}) i_1 \qquad + Z_{22} i_2,
\end{aligned}
\tag{7}
$$

where

$$Z_{11} = Z_{bb}^{(1)} + Z_{aa}^{(2)} + jX_{a1} \tag{8}$$

is the effective series impedance of the simple coaxial line consisting of conductors 1 and 2 only, and where

$$Z_{22} = Z_{bb}^{(2)} + Z_{aa}^{(3)} + jX_{a2} \tag{9}$$

is the effective series impedance of the simple coaxial line consisting of conductors 2 and 3 only. The coefficients in these equations are all per unit length of line.

In matrix notation (7) is

$$
\begin{bmatrix} \dfrac{\partial i_1}{\partial x} \\[2mm] \dfrac{\partial i_2}{\partial x} \\[2mm] \dfrac{\partial v_1}{\partial x} \\[2mm] \dfrac{\partial v_2}{\partial x} \end{bmatrix} = A' \begin{bmatrix} i_1(x) \\ i_2(x) \\ v_1(x) \\ v_2(x) \end{bmatrix}. \tag{10}
$$

This system has four eigenvalues, which are the roots of the characteristic equation of the matrix (Ref. 6, p. 314; Ref. 7, p. 111):

$$\text{determinant } [A' - \gamma I] = 0.$$

To each eigenvalue $\gamma_j (j = 1, 2, 3, 4)$ corresponds an eigenvector,

$$
\begin{bmatrix} I_j \\ V_j \end{bmatrix} = \begin{bmatrix} i_{j1}(x) \\ i_{j2}(x) \\ v_{j1}(x) \\ v_{j2}(x) \end{bmatrix}, \qquad j = 1, 2, 3, 4 \tag{11}
$$

such that

$$A' \begin{bmatrix} i_{j1} \\ i_{j2} \\ v_{j1} \\ v_{j2} \end{bmatrix} = \gamma_j \begin{bmatrix} i_{j1} \\ i_{j2} \\ v_{j1} \\ v_{j2} \end{bmatrix}, \qquad j = 1, 2, 3, 4, \tag{12}$$

where the notations

$$V(x)] = \begin{bmatrix} v_1(x) \\ v_2(x) \end{bmatrix} \qquad \text{and} \qquad \underline{V(x)} = [v_1(x), v_2(x)]$$

are used for column and row matrices respectively.

The following convention regarding subscripts will be used in the rest of this paper: If a voltage or a current bears two subscripts, the first refers to the number of the eigenvector or mode, and the second to the number of the conductor.

3.1 *Traveling Waves and Eigenvectors*

A set of certain simple solutions of the system (10) can be expressed in the form

$$\begin{aligned} i_1(x + \xi) &= \lambda i_1(x) = e^{\gamma\xi}\, i_1(x), \\ i_2(x + \xi) &= \lambda i_2(x) = e^{\gamma\xi}\, i_2(x), \\ v_1(x + \xi) &= \lambda v_1(x) = e^{\gamma\xi}\, v_1(x), \\ v_2(x + \xi) &= \lambda v_2(x) = e^{\gamma\xi}\, v_2(x). \end{aligned} \tag{13}$$

To a transmission engineer, each such solution is a *traveling wave* with a *propagation constant* γ. To a mathematician, each such solution is an *eigenvector* with an *eigenvalue* λ. More general solutions can be expressed as linear combinations of such elementary solutions. In this particular case, as we saw above, there are four traveling waves or eigenvectors, and the solutions are

$$\begin{aligned} i_1(x) &= i_{11}e^{\gamma_1 x} + i_{21}e^{\gamma_2 x} + i_{31}e^{\gamma_3 x} + i_{41}e^{\gamma_4 x}, \\ i_2(x) &= i_{12}e^{\gamma_1 x} + i_{22}e^{\gamma_2 x} + i_{32}e^{\gamma_3 x} + i_{42}e^{\gamma_4 x}, \\ v_1(x) &= v_{11}e^{\gamma_1 x} + v_{21}e^{\gamma_2 x} + v_{31}e^{\gamma_3 x} + v_{41}e^{\gamma_4 x}, \\ v_2(x) &= v_{12}e^{\gamma_1 x} + v_{22}e^{\gamma_2 x} + v_{32}e^{\gamma_3 x} + v_{42}e^{\gamma_4 x}. \end{aligned} \tag{14}$$

The eigenvalues γ_j, corresponding to the j normal traveling waves of

the system described by A', are isolated and shown clearly by factoring A' into the canonical form (Ref. 6, p. 314; Ref. 7, p. 111):

$$A' = L\Gamma L^{-1}, \tag{15}$$

where

$$\Gamma = \begin{bmatrix} \gamma_1 & 0 & 0 & 0 \\ 0 & \gamma_2 & 0 & 0 \\ 0 & 0 & \gamma_3 & 0 \\ 0 & 0 & 0 & \gamma_4 \end{bmatrix} \tag{16}$$

and

$$L = \begin{bmatrix} i_{11} & i_{21} & i_{31} & i_{41} \\ i_{12} & i_{22} & i_{32} & i_{42} \\ v_{11} & v_{21} & v_{31} & v_{41} \\ v_{12} & v_{22} & v_{32} & v_{42} \end{bmatrix} = \begin{bmatrix} I_1] I_2] I_3] I_4] \\ V_1] V_2] V_3] V_4] \end{bmatrix}. \tag{17}$$

Because A' has special properties, in particular,

$$A' = \begin{bmatrix} 0 & Y \\ Z & 0 \end{bmatrix} \tag{18}$$

the set of four equations (10) can be reduced by further differentiation to two separate sets of two equations:

$$\begin{bmatrix} \dfrac{\partial^2 i_1}{\partial x^2} \\ \dfrac{\partial^2 i_2}{\partial x^2} \end{bmatrix} = YZ \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}, \qquad \begin{bmatrix} \dfrac{\partial^2 v_1}{\partial x^2} \\ \dfrac{\partial^2 v_2}{\partial x^2} \end{bmatrix} = ZY \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}. \tag{19}$$

Because YZ is the transpose of ZY, the eigenvalues of these two systems are the same. Further, because the solutions have the forms (13) and must satisfy a relation like (12), we have

$$YZ \begin{bmatrix} i_{j1} \\ i_{j2} \end{bmatrix} = \gamma_j^2 \begin{bmatrix} i_{j1} \\ i_{j2} \end{bmatrix}. \tag{20}$$

Since the eigenvalues γ_1^2 and γ_3^2 of (19) are the squares of those of (10), we have

$$\gamma_1 = -\gamma_2 , \qquad \gamma_3 = -\gamma_4 . \tag{21}$$

They may be calculated from

$$\text{determinant } [YZ - \gamma^2 I] = 0, \tag{22}$$

as indicated above.

The ratios of magnitudes of the several traveling waves in the two conductors may be calculated from (20) and a similar equation for tbe voltages after the γ's have been found.

It may be noticed that if $Z_{ab} = 0$, we have $\gamma^2 = Y_1 Z_{11}$ or $Y_2 Z_{22}$, corresponding to two independent coaxial lines. This would be the case if the intermediate conductor tube were very thick or the frequency very high.

The above results may be extended to any number of concentric tubes. For each added tube, two more equations are added to the original set of four, and one additional distinct natural mode which may travel forward or backward appears. The number of homogeneous equations [corresponding to (20)] is one less than the number of conductors.

The computation for the three-conductor case we considered was done by machine. The results will be described in Sections V and VI.

IV. TRANSMISSION PROPERTIES OF LINES WITH TRANSPOSITIONS

In order to combine sections of uniform line and transpositions as indicated at tbe beginning of Section III, we need the relation between currents and voltages into and out of a line section of length l. This is

$$\begin{bmatrix} i_1(x + l) \\ i_2(x + l) \\ v_1(x + l) \\ v_2(x + l) \end{bmatrix} = A \begin{bmatrix} i_1(x) \\ i_2(x) \\ v_1(x) \\ v_2(x) \end{bmatrix}. \tag{23}$$

The matrix A is related to the matrix A' discussed in Section III by

$$A = L\Lambda L^{-1}, \tag{24}$$

where*

$$\Lambda = \exp(\Gamma l)$$

is a diagonal matrix with elements $\lambda_i = e^{\gamma_i l}$. Tbis throws into clear view the relation between the differential equations of the line and the trans-

* The form (23) is derived from that of (10) by considering n sections of length Δx in cascade, calculating $i(x_0 + n\Delta x)$ from $[i(x_0 + n\Delta x) - i(x_0)]/\Delta x$, and then keeping $n\Delta x = l$ while $\Delta x \to 0$. This is a case of Sylvester's theorem. A treatment is found in Ref. 7, p. 119, where polynomial functions are considered. A general proof for converging power series, including exponential series as a special case, follows by taking the limit as the sum of a polynomial series.

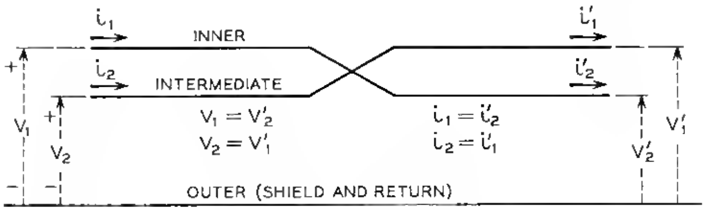


Fig. 8 — Schematic representation of a transposition in a three-conductor transmission line.

mission properties of a finite length. The eigenvectors are the same, and the eigenvalues are related through the equation

$$\lambda_i = e^{\gamma_i l}, \tag{25}$$

where $\lambda_1 = 1/\lambda_2$ and $\lambda_3 = 1/\lambda_4$ correspond to (21).

The transposition shown in Fig. 8, whose matrix we shall call T, is governed by the equations

$$\begin{aligned} v_1 &= v_2', \\ v_2 &= v_1', \\ i_1 &= i_2', \\ i_2 &= i_1', \end{aligned} \tag{26}$$

where the primed and unprimed letters refer respectively to the two sets of terminals of the six-terminal network. In matrix notation,

$$T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{27}$$

The line with transpositions is made up of a large number of equivalent sections, each containing a certain length $2l$ of transmission line and one transposition at the center, as in Fig. 7. The matrix of the transformation becomes

$$ATA = L\Lambda L^{-1}TL\Lambda L^{-1}.$$

Remember that Λ depends on the length of the section of line, although L, L^{-1} and T do not.

The characteristic equation becomes

$$\det\,[L\Lambda L^{-1}TL\Lambda L^{-1} - \nu I] = 0. \tag{28}$$

The matrix can be modified without changing its determinant by multiplying on the left by L^{-1} and on the right by L, to get

$$\det\,[\Lambda L^{-1}TL\Lambda - \nu I] = 0. \tag{29}$$

Inasmuch as the matrices Z and Y [see (7)] are both symmetrical, ZY is the transpose of YZ. Consequently, there is a set of orthogonality relations among the respective eigenvectors, as follows:

$$\underline{I_j}V_k] = i_{j1}v_{k1} + i_{j2}v_{k2} = 0$$

if

$$\gamma_j \neq \pm\gamma_k \quad \text{or} \quad \lambda_j + 1/\lambda_j \neq \lambda_k + 1/\lambda_k$$

and

$$\underline{I_j}V_k] + \underline{V_j}I_k] = 0$$

if

$$\gamma_j \neq \gamma_k \quad \text{or} \quad \lambda_j \neq \lambda_k\,.$$

The use of these relations simplifies the calculation of $L^{-1}TL$ considerably. The first step is to find L^{-1}:

$$L^{-1} = \begin{bmatrix} p_1v_{11} & p_1v_{12} & p_1i_{11} & p_1i_{12} \\ p_2v_{21} & p_2v_{22} & p_2i_{21} & p_2i_{22} \\ p_3v_{31} & p_3v_{32} & p_3i_{31} & p_3i_{32} \\ p_4v_{41} & p_4v_{42} & p_4i_{41} & p_4i_{42} \end{bmatrix} = \begin{bmatrix} p_1\underline{V_1} & p_1\underline{I_1} \\ p_2\underline{V_2} & p_2\underline{I_2} \\ p_3\underline{V_3} & p_3\underline{I_3} \\ p_4\underline{V_4} & p_4\underline{I_4} \end{bmatrix}, \tag{30}$$

where

$$p_j = \frac{1}{2\underline{I_j}V_j]}\,. \tag{31}$$

Using, in addition, the relations (21) and consequences, straightforward but tedious multiplication yields for the characteristic equation (29)

$$\begin{aligned} \nu^4 - \nu^3 Q_1(\lambda_1^2 + \lambda_2^2 - \lambda_3^2 - \lambda_4^2) \\ + \nu^2[2Q_1^2 - (Q_1^2 + Q_2^2)(\lambda_2^2\lambda_4^2 + \lambda_1^2\lambda_3^3) \\ - (Q_1^2 - Q_3^2)(\lambda_2^2\lambda_3^2 + \lambda_1^2\lambda_4^2)] \\ - \nu Q_1(\lambda_1^2 + \lambda_2^2 - \lambda_3^2 - \lambda_4^2) + 1 = 0, \end{aligned} \tag{32}$$

where

$$
\begin{aligned}
Q_1 &= (v_{12}v_{32} - v_{11}v_{31})/\Delta, \\
Q_2 &= [\sqrt{p_1/p_3}(v_{11}^2 - v_{12}^2) + \sqrt{p_3/p_1}(v_{32}^2 - v_{31}^2)]/2\Delta, \\
Q_3 &= [\sqrt{p_1/p_3}(v_{11}^2 - v_{12}^2) - \sqrt{p_3/p_1}(v_{32}^2 - v_{31}^2)]/2\Delta, \\
\Delta &= v_{11}v_{32} - v_{12}v_{31} .
\end{aligned}
$$

Equation (31) defines p_1 and p_3, and

$$Q_1^2 + Q_2^2 - Q_3^2 = 1.$$

As anticipated, the equation has symmetrical coefficients, which means that the roots occur in symmetrical pairs, and also means that a substitution $\nu + 1/\nu = \xi$ reduces the degree to two. The roots ν_i of the characteristic equation (32) are the eigenvalues of the system and are related to the effective propagation constants γ_{Ti} of a transposed line by $\nu_i = e^{\gamma_{Ti} 2l}$. By this means, the propagation constants of the transposed line were derived from the eigenvectors and propagation constants of a section without transposition, with the aid of matrix properties of the line. The actual numerical computation was performed on a large electronic computer for several different transposition intervals, and the results are given both graphically and in tabular form in Sections V and VI.

4.1 *Infinitesimal Spacing of Transpositions*

A special case of the general results just given yields results with much less numerical calculation and throws interesting light on the whole subject. From it, a very simple picture of the action of transposed lines is derived.

Consider the situation when the uniform line sections on either side of a transposition have length dx instead of l. The matrix A now may be formed from A' by calculating $i_1(x + dx)$, etc., from di_1/dx. In this way we get

$$
A = \begin{bmatrix}
1 & 0 & Y_1\,dx & -Y_1\,dx \\
0 & 1 & -Y_1\,dx & (Y_1 + Y_2)\,dx \\
(Z_{22} + Z_{11} - 2Z_{ab})\,dx & (Z_{22} - Z_{ab})\,dx & 1 & 0 \\
(Z_{22} - Z_{ab})\,dx & Z_{22}\,dx & 0 & 1
\end{bmatrix}.
$$

From this, the matrix ATA for an element of transposed line is obtained easily. But, for present purposes, it is desirable to maintain the positions of $i_1(x)$, $i_2(x)$, etc., in the column matrices of (23), and so we will consider a unit of length $4\,dx$ having a matrix $ATAATA$.

Just as was found in Section III, it is easier to handle a system like (10) with matrix A' when the line is divided into equivalent sections of infinitesimal length. To do this, $4\partial i_{T1}/\partial x$ is calculated from $[i_{T1}(x + 4dx) - i_{T1}(x)]/dx$, etc. The transposed line composed of these very short elements is described, as in (10), by

$$\begin{bmatrix} \dfrac{\partial I_T]}{\partial x} \\ \dfrac{\partial V_T]}{\partial x} \end{bmatrix} = A_T' \begin{bmatrix} I_T] \\ V_T] \end{bmatrix}, \tag{33}$$

where

$$A_T' = \begin{bmatrix} 0 & Y_T \\ Z_T & 0 \end{bmatrix}, \qquad Y_T = \begin{bmatrix} 2Y_1 + Y_2/2 & -Y_1 \\ -Y_1 & 2Y_1 + Y_2/2 \end{bmatrix},$$

and

$$Z_T = \begin{bmatrix} Z_{22} + Z_{11}/2 - Z_{ab} & Z_{22} - Z_{ab} \\ Z_{22} - Z_{ab} & Z_{22} + Z_{11}/2 - Z_{ab} \end{bmatrix}.$$

The eigenvalues γ_{Tj} of (33) occur in pairs and are calculated from determinant $[Y_T Z_T - \gamma_T^2 I] = 0$. The eigenvectors $I_{Tj}]$ and $V_{Tj}]$ are calculated from

$$Y_T Z_T \begin{bmatrix} i_{Tj1} \\ i_{Tj2} \end{bmatrix} = \gamma_{Tj}^2 \begin{bmatrix} i_{Tj1} \\ i_{Tj2} \end{bmatrix}, \text{ etc.,} \tag{34}$$

as before. The results are

(Low loss mode)

$$\begin{aligned} \gamma_{T1}^2 &= Y_2(Z_{22} - Z_{ab} + Z_{11}/4), \qquad \gamma_{T2} = -\gamma_{T1}, \\ i_{11} &= i_{12} = v_{11} Y_2/2\gamma_{T1} = v_{12} Y_2/2\gamma_{T1}, \\ i_{21} &= i_{22} = v_{21} Y_2/2\gamma_{T2} = v_{22} Y_2/2\gamma_{T2}, \end{aligned} \tag{35}$$

(High loss mode)

$$\begin{aligned} \gamma_{T3}^2 &= (Y_1 + Y_2/4) Z_{11}, \qquad \gamma_{T4} = -\gamma_{T3}, \\ i_{31} &= -i_{32} = v_{31}(4Y_1 + Y_2)/2\gamma_{T3} = -v_{32}(4Y_1 + Y_2)/2\gamma_{T3}, \\ i_{41} &= -i_{42} = v_{41}(4Y_1 + Y_2)/2\gamma_{T4} = -v_{42}(4Y_1 + Y_2)/2\gamma_{T4}, \end{aligned}$$

where the T subscript on the voltage and current components has been dropped for simplicity.

It is clear in this case that one mode has the expected current distribution, i.e., $i_{11} = i_{12}$, in which equal currents flow in the two inner conductors. This confirms the heuristic analysis in an earlier section, where we assumed that such a current distribution could be achieved and conjectured that it could be achieved by transposition at sufficiently short intervals. A detailed comparison with the estimate of loss L in Section 2.2 shows that the predicted loss is the same as estimated before. However, the present analysis is more revealing, because it not only tells the loss of the desired mode, but also shows the losses and current and voltage distributions of both modes. A knowledge of the current and voltage distributions is necessary to launch one mode without exciting the other. In this case, connecting the two inner conductors together, so that $v_1 = v_2$ at the launching point, is enough to guarantee that the unwanted mode cannot be excited, for in the unwanted mode $v_{31} - v_{32}$ does not equal zero.

4.2 *Simple Picture of Attenuation in the Transposed Line*

An interesting picture of the operation of the transposed three-conductor line can be obtained from (35) for γ_{T1}^2 when the transposition interval is very small, (γ_{T1} being the propagation constant for the low-loss mode). In this expression, $Y_2 = jS_2$ may be taken as the radial admittance and $Z_{22} - Z_{ab} + Z_{11}/4$ as the series impedance of an equivalent two-conductor coaxial. If the series impedance is $R + jX$ and R is very much less than X, we have approximately

$$\gamma_{T1} = \frac{R}{2}\sqrt{\frac{S_2}{X}} + j\sqrt{S_2 X}.$$

The attenuation (r.p. γ_T) is proportional to the resistance of the series impedance, and we will consider only this:

$$\begin{aligned} R &= R_{22} - R_{ab} + R_{11}/4 \\ &= R_{aa}^{(3)} + R_{bb}^{(2)} - R_{ab} + \frac{R_{aa}^{(2)}}{4} + \frac{R_{bb}^{(1)}}{4}. \end{aligned}$$

Here, $R_{aa}^{(3)}$ is the resistance in ohms per meter of length of the outer conductor; $R_{bb}^{(2)}$ and $R_{aa}^{(2)}$ are the surface resistances along the outer and inner surfaces of the intermediate tube; R_{ab} is the transfer resistance of this tube and $R_{bb}^{(1)}$ is the surface resistance of the inner solid conductor.

Because the intermediate tube is electrically thin for the frequencies of interest, $R_{bb}^{(2)}$ and R_{ab} are quite flat as functions of frequency and very nearly cancel each other. Eventually, $R_{bb}^{(2)}$ begins to increase and R_{ab} to decrease with frequency. Until this frequency is reached, the re-

sistance of the central conductor of the equivalent two-conductor coaxial is given very closely by

$$R_{c3} = \tfrac{1}{4}[R_{aa}{}^{(2)} + R_{bb}{}^{(1)}].$$

A physical picture of the hypothetical center conductor which agrees with this expression is shown in Fig. 9 for a line n meters long. This has a real correspondence to the actual transposed line in that each of the two inner conductors consists of alternate sections of intermediate and inner conductors having the resistances per meter of the two terms above, and each carries approximately the same value of current. Of course, the transposition interval is not necessarily one meter, as indicated in the figure. Fig. 9 also shows a representation of the central solid conductor of the reference two-conductor coaxial. It is assumed here that the intermediate tube and its inner insulation are very thin, so that their combined diameters are essentially the same as the diameter of the solid reference conductor. Its resistance per meter is then $R_{c2} = R_{bb}{}^{(1)}$.

Formulae for calculating the surface impedances are given in Section VI. The first terms of each of the approximations (38) are suitable for

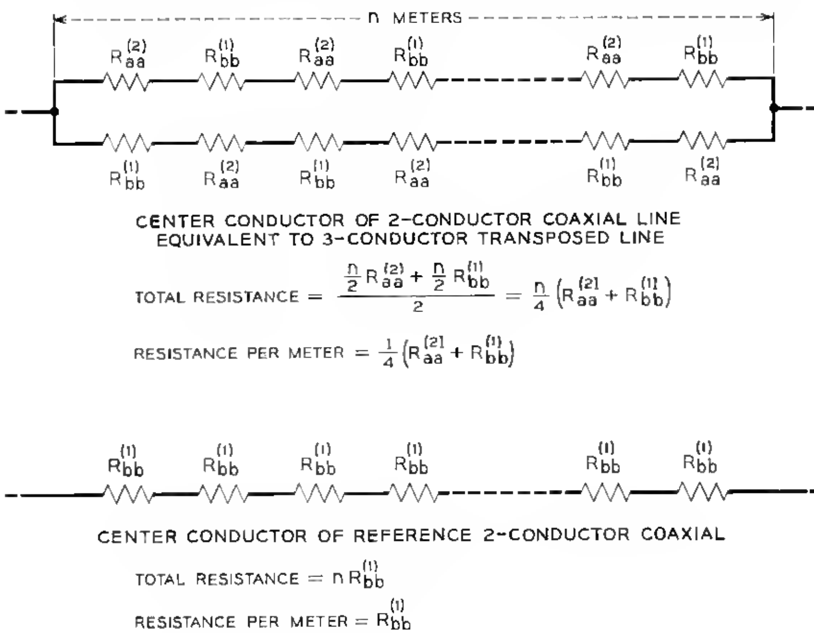


Fig. 9 — Low-frequency representation of inner and intermediate conductors in three-conductor transposed line.

present purposes and are reliable even at low frequencies for a thin-walled tube. Thus

$$Z_{aa}^{(2)} = \frac{\eta}{2\pi a_2} \frac{1}{\sigma t} = R_{aa}^{(2)} = \frac{1}{2\pi a_2 g t},$$

$$R_{bb}^{(1)} = \frac{1}{2b_1} \sqrt{\frac{\mu f}{\pi g}},$$

using (37) also. If, in addition, b_2, a_2 and b_1 are approximately equal,

$$\frac{R_{aa}^{(2)}}{R_{bb}^{(1)}} = \frac{\delta}{t},$$

where

$$\delta = \sqrt{\frac{1}{\pi \mu g f}}$$

is the skin depth and t is the thickness of the intermediate conductor. Then, the ratio of center conductor resistance in the three-conductor transposed line to that of the two conductor reference line is

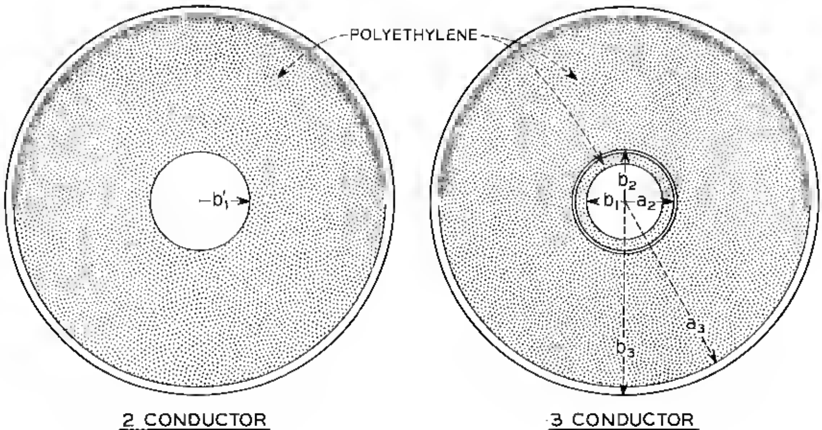
$$K = \frac{R_{c3}}{R_{c2}} = \frac{1}{4}\left(\frac{\delta}{t} + 1\right).$$

At the frequency for which skin depth equals intermediate conductor thickness, the resistance is cut in half. If 78 per cent of the cable loss occurs in the center conductor, a 39 per cent reduction of attenuation would result. This is quite a bit more than was found experimentally. The difference is that, even for $\delta/t = 1$, R_{ab} does not cancel $R_{bb}^{(2)}$ exactly. If the third terms in coth σt and csch σt (the second terms give reactance only) are added to the above simple approximations, we find

$$K = \frac{R_{c3}}{R_{c2}} = \frac{1}{4}\left(\frac{\delta}{t} + 1\right) + \frac{1}{6}\left(\frac{t}{\delta}\right)^3.$$

Now, when δ/t is 1, K equals 0.667 and a 33 per cent reduction of resistance or 25.8 per cent reduction of attenuation is obtained. This is very close to the value computed from the whole expression for one-mil polyethylene (Fig. 18). When the cubic term is appreciable, the simple picture of Fig. 9 is no longer sufficient.

We find K equals one for $\delta/t = 3$ and $\delta/t = 0.66$. The first is the lower-frequency crossover point (0.54 mc) and the second is the higher cross-over point (11.1 mc) where the three-conductor transposed line has the same loss as the reference two-conductor line. These values agree quite



	OUTSIDE DIAMETER	THICKNESS
OUTER	0.180″	0.004″
INNER	0.047″	

	OUTSIDE DIAMETER	THICKNESS
OUTER	0.180″	0.004″
INTERMEDIATE	0.049″	0.0012″
INNER	0.0355″	

Fig. 10 — Cross sections of two-conductor and three-conductor coaxial lines used in experiment.

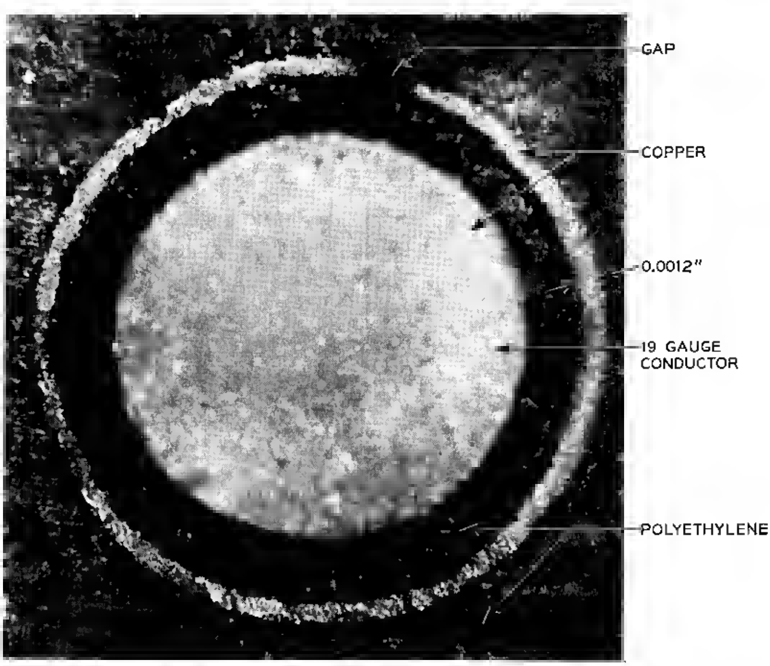


Fig. 11 — Photomicrograph of cross section of the inner conductors of the three-conductor coaxial line.

well with those calculated from the full expression for attenuation and plotted on Fig. 18.

The above simple expression for center conductor resistance reduction thus gives a good description of three-conductor transposed line performance, provided the radii of the two conductors are nearly the same.

If two thin shells surround an inner conductor to make a four-conductor transposed line, the simple picture of Fig. 9 may be generalized to give

$$K = \frac{R_{c4}}{R_{c2}} = \frac{1}{9}\left(2\frac{\delta}{t} + 1\right)$$

for low frequencies. At $\delta/t = 1$, this would yield a 67 per cent resistance reduction or 52 per cent attenuation reduction. Naturally the cubic term is bigger than for the three-conductor line, and its effect would be felt at lower frequencies.

V. EXPERIMENT AND COMPARISON WITH THEORY

As mentioned above, it was predicted that there would be a reduction of attenuation in a coaxial line whose center conductor was laminated, with the laminations transposed periodically along the line. Confirmation of this prediction by resistance measurements of a three-foot length of such a line led to the decision to test the idea more fully by transmission measurements on a long section of line.

5.1 *Making the Cable*

This required the manufacture of about a mile of three-conductor coaxial cable of unusual design. A design and a method of fabrication were worked out in cooperation with members of the Western Electric Company. Fig. 10 shows a cross section of the finished cable and its dimensions. The inner conductor is 19-gauge copper wire over which polyethylene was extruded to a thickness of 5 mils. Using a special die, the intermediate conductor was made by forming a copper tape about 1 mil thick and about 150 mils wide over the insulated wire, leaving a longitudinal seam. Ordinarily such a seam must be held closed with a helical wrapping of some kind, but this was found unnecessary when the second and comparatively thick layer of polyethylene dielectric was extruded over this intermediate conductor immediately after it had been formed by the die. One factor contributing to the success of this operation was the leaving of a small gap in the seam rather than an overlap. Microscopic examination of cross sections (see Fig. 11) showed the gap to be

about 5 mils or less. Capacitance measurements also indicated the presence of a slight air gap a few tenths of a mil thick between the inner and intermediate conductors. The final operation consisted of forming the outer conductor, using a copper tape 4 mils thick by 0.600 in. wide and a special forming tool. In this case, there is a small overlap at the longitudinal seam, which is held closed by a helical wrapping of paper tape, 5 mils by 0.750 in., applied 20 wraps per foot of cable. The effective air gap between the intermediate and outer conductors is about 1 mil, as determined by capacitance measurements.

At the same time this cable was made, a two-conductor coaxial cable was also made for comparison purposes. The central conductor of this cable is a solid copper wire of 0.0470-in. diameter, which is just a little less than the outside diameter of the intermediate conductor of the other cable. For both cables, the polyethylene dielectric was extruded through the same die and the outer conductor was made from the same batch of copper and applied in the same way. A cross section with dimensions for this cable is shown in Fig. 10. Sections of both cables are shown in Fig. 12.

The use of a reference cable made in the way described makes it unnecessary for the testing of the attenuation reduction idea to depend on

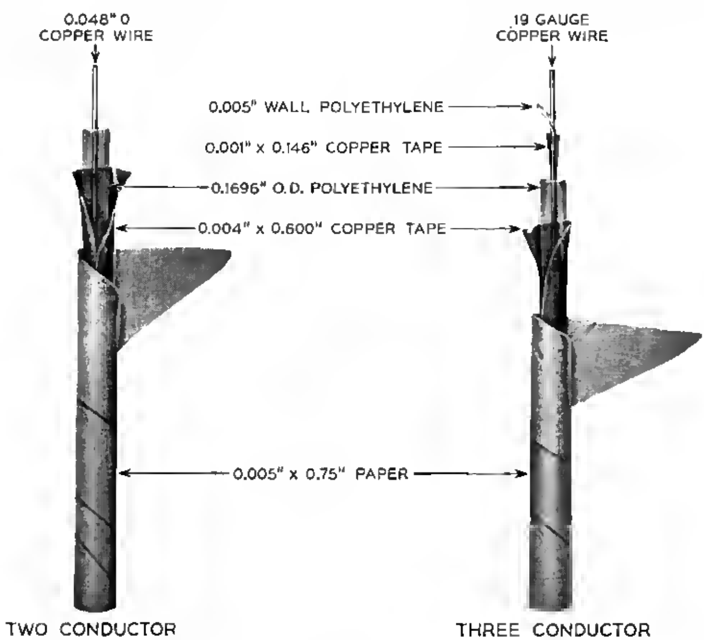


Fig. 12 — Photographs of two-conductor and three-conductor coaxial lines used in experiment.

a comparison of measurement with calculation. Rather, it depends on the comparison of the measured attenuations of two similar real cables, one having a solid central conductor, the other having a laminated central conductor in which the laminations are transposed regularly.

Open and short-circuit impedance measurements showed that the characteristic impedance of the three-conductor cable is very near 50 ohms. According to the estimates of Section II, the frequency at which maximum reduction of attenuation occurs is determined by the thickness of the intermediate conductor. For the cable described above, this is about 4 mc.

Because of the way in which they are made, the intermediate and outer conductors will buckle or even break if the cable is bent with too small a radius. To insure further against possible buckling or breaking, it was planned to handle the cable as little as possible. Hence, it was decided to re-reel the cable in a single layer onto suitable drums as the transpositions were made, so that access to any part of the cable would be possible without additional unreeling and re-reeling.

5.2 *Making the First Transpositions*

After the cable was fabricated as described above, the major part of the laboratory work consisted of devising and making satisfactory transpositions between the inner and intermediate conductors at regular intervals. At a point where a transposition was desired, the cable was cut and conductors and dielectric were stripped back so that all three conductors were laid bare at both sides of the cut. By means of special jacks and a plug, the center conductor on the left side of the cut was connected to the intermediate conductor on the right side of the cut and the intermediate conductor on the left side was connected to the center conductor on the right side. The two outer conductors were then connected together. Fig. 13 shows the cable ends connected to the jacks.

Because of the fragile nature of the intermediate conductor and its supporting insulation, considerable care had to be exercised in making the transpositions. Ragged edges on the 1-mil copper had to be guarded against to avoid puncturing the 5-mil polyethylene. Damage to the insulation from excessive heat was avoided by using low-melting-point solder for making electrical connection to the intermediate conductor.

5.3 *First Measurements, Showing the Effect of Too Long a Transposition Interval*

At the two ends of the cable, the inner and intermediate conductors were connected together to serve as one input and output terminal. Fifty-

ohm terminations were used. The results of the transmission measurements are plotted on Fig. 14, along with those for the reference two-conductor coaxial. These were disappointing because, in the frequency region where a reduction of loss was expected, two large peaks of increase were found. Measurements made after changing all the patching plugs for straight-through connection gave results very nearly the same and very nearly as smooth as those obtained from a continuous piece of cable of the same length. Apparently the transpositions and not the discontinuity at the section junctions were responsible for the peaks. With only one transposition at the center of the 1180-foot section, there were ±0.2-db ripples in the loss curve. The effects of reflection at the uniformly spaced transpositions accumulated, with the results already seen. Investigation showed the transposition interval to be one-quarter wavelength at the first peak and three-quarters of a wavelength at the second. Spacing the transpositions twice as far apart lowered the frequencies of the peaks by a factor of two. If there was indeed a reduction of attenuation between 1 and 10 mc, it was covered up by the reflection effects.

At this point, a thorough calculation of transmission loss was begun to help understand the phenomenon. Further experimental work was undertaken at the same time. Since no peaks appeared at frequencies below that corresponding to an interval of one-quarter wavelength, it seemed reasonable to expect the looked-for reduction of attenuation if the transposition interval was shortened so that the frequency for which it was one-quarter wavelength was outside the desired frequency region. Using our previous experience as a guide, a spacing of about 9 feet was chosen. With this spacing, the first peak should have been at around 20 mc.

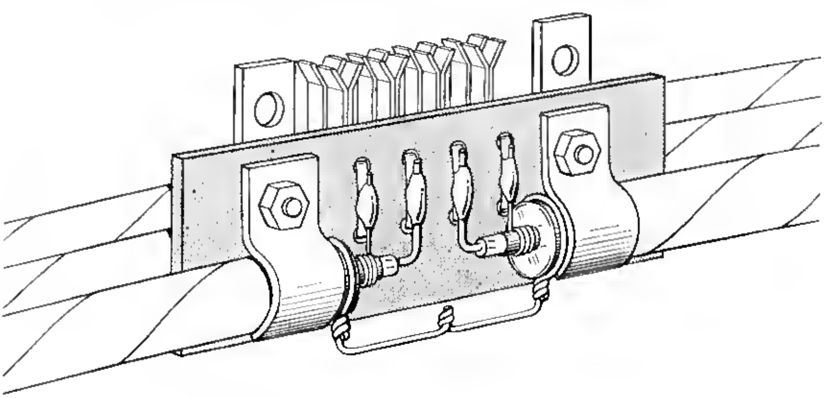


Fig. 13 — Technique for making cable connections at a plug-in transposition point.

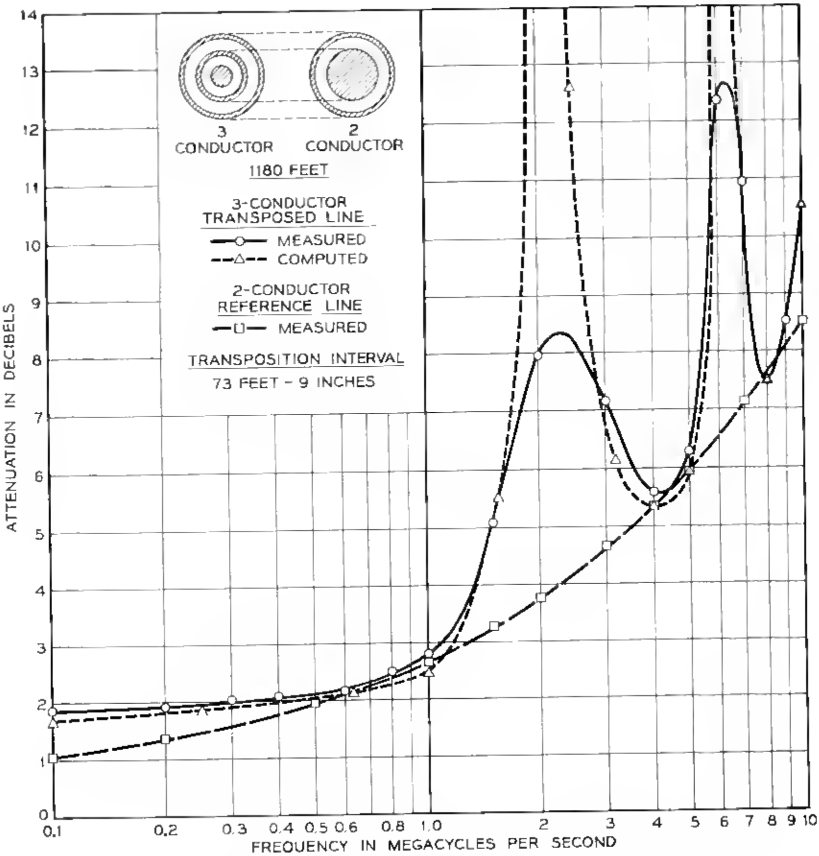


Fig. 14 — Attenuation of line with 75-foot transposition spacing (approx.) compared with that of reference two-conductor line and with computations.

5.4 *Final Measurement and Comparison with Theory*

The plug-in transposition in Fig. 13 was too elaborate to be used in such large numbers, and not too satisfactory. Therefore, a permanent transposition, shown in Fig. 15, was devised. A continuous cable having 113 sound transpositions and 114 sections in a total length of 1057 feet was made ready for test. This time, after transmission measurements had been made and compared with those for the two-conductor reference line, the expected reduction of attenuation was found, as shown in Fig. 16. The reduction at 4 mc in the center of the region is 17 per cent.

While the rough estimate of reduction made in Section II was 27 per cent, it should be pointed out that this depended on the assumption that

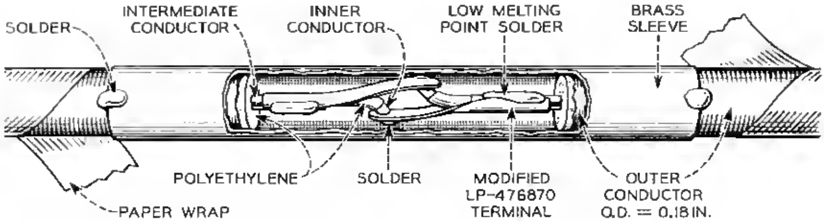


Fig. 15 — Cutaway illustration of permanent transposition.

the diameter of the solid inner conductor be substantially the same as that of the tubular intermediate conductor. In the actual cable, the diameter of the solid core is 28 per cent less than that of the surrounding tube, mainly because of the 5-mil polyethylene insulator. Therefore, a reduction of only 15 or 20 per cent should be expected. The actual reduction is right in the middle of this range.

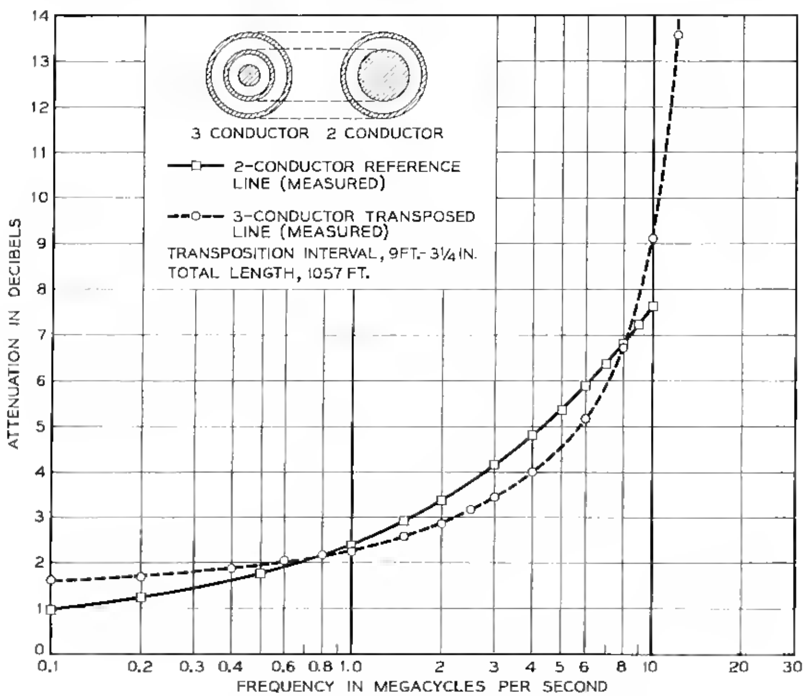


Fig. 16 — Attenuation of line with 9-foot transposition spacing (approx.) compared with that of the reference two-conductor line.

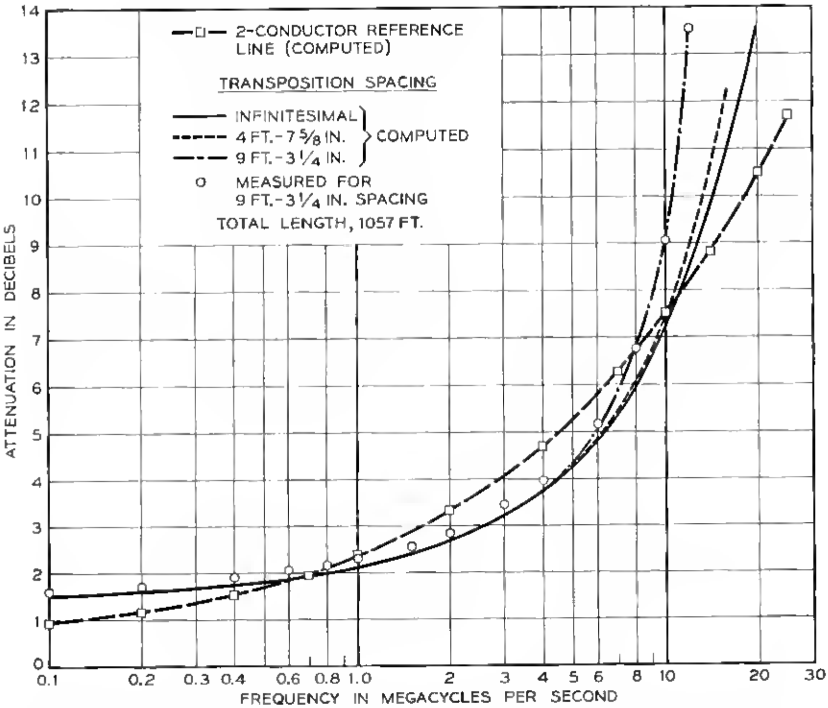


Fig. 17 — Calculation of attenuation with transpositions at approx. 9 feet, approx. $4\frac{1}{2}$ feet and very small intervals; comparison with measurements where available.

The calculation of transmission loss described in Section IV was finished after the measurements. The results are plotted on Fig. 17 and show very good agreement with the measured data.

The greater attenuation at very low frequencies is caused mainly by the transpositions and not by the fact that the central conductor cross section is 38 per cent less in the three-conductor cable than it is in the two-conductor one. The central conductor of the transposed line at dc consists of two like structures in parallel. Each of these is a conductor of many series sections, with inner and intermediate sections alternating. Thus, the considerably higher-resistance intermediate sections dominate the situation. At very low frequencies, the attenuation ratio is approximately 2, which is slightly greater than the value 1.6 shown at 100 kc.

5.5 *Transposition Interval*

In the calculation it was very easy to shorten the transposition interval. Two such cases are plotted in Fig. 17. When the spacing is just one-

half as long as before, the loss curve is nearly the same, but it crosses over the reference cable loss at about 10 mc instead of at 8 mc. Thus, the reflection peak at about 20 mc (not shown on the figure) pulls up the loss curve a little at the high end. The other case calculated was for infinitesimally small spacing of transpositions, and this gave substantially the same loss curve as the $4\frac{1}{2}$-foot spacing. We conclude that the reflection peak at 40 mc has a negligible effect on the loss curve in the region of improvement. Also, it is seen that the 17 per cent improvement observed experimentally would be raised to 18 or 19 per cent by spacing the transpositions at about 4-foot intervals.

The attenuation was calculated also for the cable with transpositions 75 ft 9 in. apart. The results, in good agreement with the measurements, are plotted on Fig. 14. While a random longer spacing of transpositions would greatly reduce the sharp peaks of loss in Fig. 14, the way to get the smoothest loss curve with the greatest reduction of loss is to make

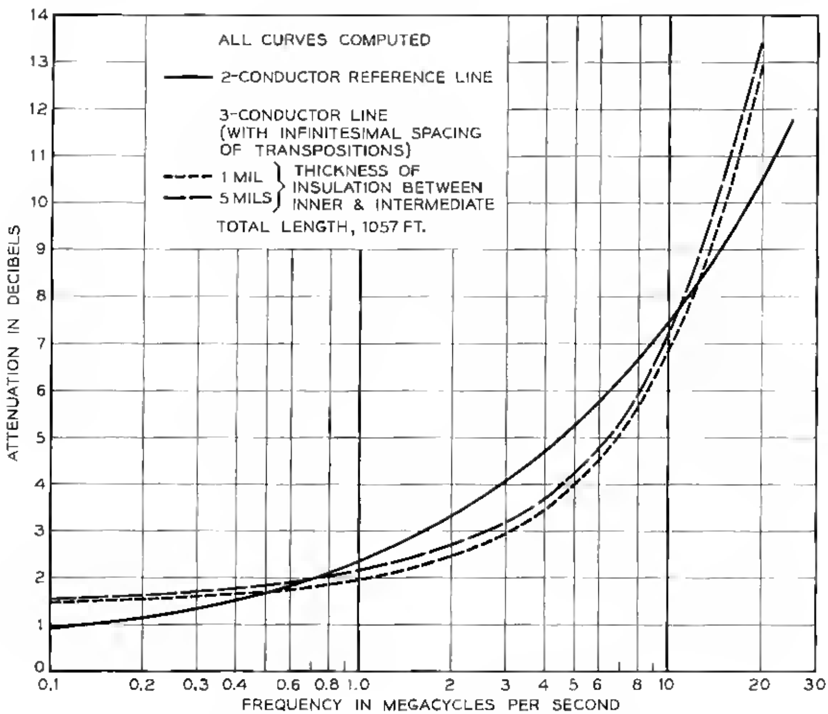


Fig. 18 — Calculated attenuation of a transposed line similar to that used in experiments, but with 1-mil dielectric thickness between innermost conductors.

the spacing small enough so that the first peak is well beyond the desired operating region.

5.6 *Effect of Spacing Between Inner and Intermediate Conductors*

Another result should be of interest here even though it was not obtained experimentally. A calculation for loss was made for a line in which the inner conductor was enlarged so that the polyethylene separating it from the intermediate conductor was only 1 mil thick instead of 5 mils. The results are plotted on Fig. 18. All curves are calculated so that they are on the same basis. At 4 mc, the transposed three-conductor with 5-mil polyethylene shows an improvement of 21.2 per cent, and the transposed three-conductor with 1-mil polyethylene shows an improvement of 26.9 per cent.

The latter case approximates the original assumption of equal diameters much more closely than does the former, and shows an improvement very nearly the same as the 27 per cent prediction of Section II. It was realized in the beginning that the dielectric separating the inner and intermediate conductors should be as thin as possible, but 5 mils was a much more practical figure both for fabrication of the cable and for the transpositions. These results serve to point the way to further improvements.

5.7 *Conclusions*

The experiments have compared the transmission loss of two coaxial cables which are the same except that the center conductor of one is solid while that of the other consists of a shell surrounding and insulated from a center core. It has been shown that the transmission loss is less, over a certain frequency band, for the cable with the laminated center conductor, provided the laminations are transposed at sufficiently frequent intervals. Also, calculations of transmission loss show very good quantitative agreement with measured values, thus tieing the theory to the practice. Further, the results show that, if the insulating space between intermediate and inner conductors can be reduced to a thickness negligible compared to its radius, the reduction of loss can be greater and will apply over a greater frequency band.

VI. NUMERICAL CALCULATION

6.1 *Surface Impedances and Propagation Constants for Uniform Line*

The conductor surface impedances referred to in Sections II and III may be calculated from formulae given in Schelkunoff's paper.[2]

When the calculations were started, no attenuation reduction had been found experimentally, even though measurements had been made. Hence, we wanted to make the calculations as exact as possible in order that no small effect in the uniform line properties, which might have a large effect in the transposed line, would be overlooked. Therefore, tbe exact expressions for the surface impedances [equations (75) in Ref. 2] were used as far as possible. Expressed in terms of modified Bessel functions of the first and second kind, I_n and K_n, respectively, the results are:

$$\begin{aligned}
Z_{bb}^{(1)} &= \frac{\eta}{2\pi b_1} \frac{I_0(\sigma b_1)}{I_1(\sigma b_1)}, \\
Z_{aa}^{(2)} &= \frac{\eta}{2\pi a_2} \frac{I_0(\sigma a_2)K_1(\sigma b_2) + K_0(\sigma a_2)I_1(\sigma b_2)}{I_1(\sigma b_2)K_1(\sigma a_2) - I_1(\sigma a_2)K_1(\sigma b_2)}, \\
Z_{bb}^{(2)} &= \frac{\eta}{2\pi b_2} \frac{I_0(\sigma b_2)K_1(\sigma a_2) + K_0(\sigma b_2)I_1(\sigma a_2)}{I_1(\sigma b_2)K_1(\sigma a_2) - I_1(\sigma a_2)K_1(\sigma b_2)}, \\
Z_{ab}^{(2)} &= \frac{1}{2\pi g a_2 b_2} \frac{1}{I_1(\sigma b_2)K_1(\sigma a_2) - I_1(\sigma a_2)K_1(\sigma b_2)}, \\
Z_{aa}^{(3)} &= \frac{\eta}{2\pi a_3} \frac{I_0(\sigma a_3)K_1(\sigma b_3) + K_0(\sigma a_3)I_1(\sigma b_3)}{I_1(\sigma b_3)K_1(\sigma a_3) - I_1(\sigma a_3)K_1(\sigma b_3)}.
\end{aligned} \tag{36}$$

In these expressions, the b's and a's are the radii of the various conductor surfaces as shown on Fig. 1, and η and σ are the intrinsic impedance and the propagation constant of the conductor material. Tbey are

$$\begin{aligned}
\eta &= (1 + i) \sqrt{\frac{\pi f \mu}{g}}, \\
\sigma &= (1 + i) \sqrt{\pi f \mu g}, \\
i &= \sqrt{-1}.
\end{aligned}$$

The dimensions of the cables as supplied by Western Electric are:

$$\begin{aligned}
b_1 &= 0.0178 \text{ in.} = 4.52 \times 10^{-4} \text{ meter}, \\
a_2 &= 0.0234 \text{ in.} = 5.95 \times 10^{-4} \text{ meter}, \\
b_2 &= 0.0246 \text{ in.} = 6.25 \times 10^{-4} \text{ meter}, \\
a_3 &= 0.0862 \text{ in.} = 2.19 \times 10^{-3} \text{ meter}, \\
b_3 &= 0.0902 \text{ in.} = 2.29 \times 10^{-3} \text{ meter}.
\end{aligned}$$

For the two-conductor reference line, a_3 and b_3 are the same as above and

$$b_2' = 0.0235 \text{ in.} = 5.97 \times 10^{-4} \text{ meter}.$$

The cable material constants as supplied by the makers of the cable are

$$\mu = \mu_0 = 4\pi \times 10^{-7} \text{ henry/meter}$$

$$\epsilon = 2.2\epsilon_0 = \frac{2.2}{36\pi} \times 10^{-9} \text{ farad/meter}$$

$$g = 5.858 \times 10^{7} \text{ mho/meters}$$

(at 68°F this is 101 per cent IACS).

These impedances were calculated by means of the IBM 650 computer from 100 kc to several megacycles at points uniformly spaced on a logarithmic frequency scale. Each frequency point is higher than the one preceding it by a factor of $10^{0.1}$. In the main program, the libraries used for computing the modified Bessel functions at each point were prepared by Miss M. C. Gray.

For sufficiently high frequencies, approximate formulas for the surface impedances are satisfactory. These are:

$$\begin{aligned}
Z_{bb}^{(1)} &= \frac{\eta}{2\pi b_1}\left[1 + \frac{1}{2\sigma b_1}\right], \\
Z_{aa}^{(2)} &= \frac{\eta}{2\pi a_2}\left[\coth \sigma(b_2 - a_2) - \frac{1}{8\sigma}\left(\frac{1}{a_2} + \frac{3}{b_2}\right)\right. \\
&\qquad \left. - \frac{3}{8\sigma}\left(\frac{1}{a_2} - \frac{1}{b_2}\right)\coth^2 \sigma(b_2 - a_2)\right], \\
Z_{bb}^{(2)} &= \frac{\eta}{2\pi b_2}\left[\coth \sigma(b_2 - a_2) + \frac{1}{8\sigma}\left(\frac{3}{a_2} + \frac{1}{b_2}\right)\right. \\
&\qquad \left. - \frac{3}{8\sigma}\left(\frac{1}{a_2} - \frac{1}{b_2}\right)\coth^2 \sigma(b_2 - a_2)\right], \\
Z_{ab} &= \frac{\eta}{2\pi\sqrt{a_2 b_2}} \frac{\left[1 - \dfrac{3}{8\sigma}\left(\dfrac{1}{a_2} - \dfrac{1}{b_2}\right)\coth \sigma(b_2 - a_2)\right],}{\sinh \sigma(b_2 - a_2)} \\
Z_{aa}^{(3)} &= \frac{\eta}{2\pi a_3}\left[\coth \sigma(b_3 - a_3) - \frac{1}{8\sigma}\left(\frac{1}{a_3} + \frac{3}{b_3}\right)\right. \\
&\qquad \left. - \frac{3}{8\sigma}\left(\frac{1}{a_3} - \frac{1}{b_3}\right)\coth^2 \sigma(b_3 - a_3)\right].
\end{aligned} \tag{38}$$

The higher-order correction terms are not the same as the ones given by Schelkunoff's paper[2] which, as pointed out by Miss Gray, are incorrect. These also were computed using the IBM 650 machine. However, they went through much faster than the exact ones, and so all the fre-

quency points (25) from 100 kc to 25 mc were computed. As it turned out, the approximations were very close to the exact values of impedance except at the lowest few frequency points, and the greatest differences were in the reactances.

The resistance and reactance of $Z_{bb}^{(1)}$ (surface impedance of the inner conductor) are fairly close to one another, increasing about as $\sqrt{f}$ and having values of 0.1 ohm near 1 mc. The three resistances $R_{aa}^{(2)}$, $R_{bb}^{(2)}$ and R_{ab} of the intermediate conductor are nearly the same and constant at about 0.15 ohm up to about 3 mc, where the first two begin to increase and R_{ab} begins to decrease. The corresponding reactances vary linearly with frequency over most of the range, the first two being positive and equal to the resistance at about 0.7 mc, and X_{ab} being negative and equal to R_{ab} at about 1.5 mc. The resistance $R_{aa}^{(3)}$ begins to increase from 0.015 ohms around 0.4 mc. All these values are per meter of length.

In the third calculation using the IBM 650, the inductive reactances X_{a1} and X_{a2} (5) of the space between conductors were computed and combined with the proper values of surface impedances at each of the 25 frequency points (exact as far as they went and approximate beyond) to give the series impedances Z_{11} and Z_{22}, (8) and (9). The dielectric admittances Y_1 and Y_2 (6) and the transfer impedance Z_{ab} were then combined with these to form the coefficients of the characteristic equation (22). The four roots (two distinct) of this equation are the propagation constants of the uniform three-conductor coaxial line. Voltage and current ratios were computed at each mode, using (20). These quantities are shown in Table I.

The numbers tabulated in Table I contain eight significant digits, just as they came from the IBM 650 computer. This precision would not be justified by the primary data if these were final results. But they are intermediate results which were used as input data to the computer for the transposed line calculation.

6.2 *Transposed Line*

From the characteristic equation (32) for the transposed line section of length $2l$ having one transposition at its center, the four roots ν_i were found and, from these, the four (two distinct) propagation constants γ_{Ti} were calculated. This was done for the transposition interval $2l$ = 73 ft 9 in. of the first experiment and the interval $2l$ = 9 ft $3\frac{1}{4}$ in. of the second experiment, and for an interval of 4 ft $7\frac{5}{8}$ in. not done experimentally. Only the λ_i vary with transposition interval $2l$, since the Q's depend only on current and voltage ratios of the natural travelling waves. These quantities are shown in Table II.

TABLE I — UNIFORM THREE-CONDUCTOR LINE — $\gamma_1 = \alpha_1 + j\beta_1$

Frequency, mc	$10^4\alpha_1$ nepers/meter	$10^2\beta_1$ radians/meter
0.10000000	3.4688831	0.33348735
0.15848935	4.0799877	0.52214634
0.25118873	4.8395041	0.81834244
0.39810738	5.7671483	1.2854328
0.63095778	6.9660247	2.0228904
1.0000008	8.5580372	3.1876156
1.5848947	10.693456	5.0275308
2.5118893	13.462726	7.9349729
3.9810768	16.887865	12.532983
6.3095828	21.135279	19.809733
10.000016	26.526402	31.329134
15.848960	33.384128	49.567992
25.118912	42.047592	78.451609

Frequency, mc	$10^3\alpha_3$ nepers/meter	$10^2\beta_3$ radians/meter
0.10000000	4.7318826	0.62320343
0.15848935	5.6942080	0.84536090
0.25118873	6.7525699	1.1723877
0.39810738	7.8427964	1.6701939
0.63095778	8.9337375	2.4480157
1.0000008	10.044322	3.6761608
1.5848947	11.241957	5.6192649
2.5118893	12.643512	8.6900187
3.9810768	14.433451	13.536061
6.3095828	16.928777	21.174433
10.000016	20.664341	33.195442
15.848960	26.361051	52.069905
25.118912	34.329361	81.679447

Frequency, mc	i_{12}/i_{11} (r.p.)	i_{12}/i_{11} (i.p.)
0.10000000	0.18362010	0.16630834
0.15848935	0.22474330	0.22944720
0.25118873	0.27695130	0.30501473
0.39810738	0.33751280	0.40492969
0.63095778	0.40881000	0.54361980
1.0000008	0.48911890	0.73916345
1.5848947	0.56956110	1.0221795
2.5118893	0.62801670	1.4417420
3.9810768	0.60888470	2.0726476
6.3095828	0.37242140	3.0234415
10.000016	−0.41964610	4.4235045
15.848960	−2.5566098	6.2815904
25.118912	−7.7731939	7.8207114

Frequency, mc	i_{32}/i_{31} (r.p.)	i_{32}/i_{31} (i.p.)
0.10000000	−1.1816245	0.025519684
0.15848935	−1.1729224	0.032395813
0.25118873	−1.1624073	0.038792862
0.39810738	−1.1501389	0.045454252
0.63095778	−1.1354390	0.052262004
1.0000008	−1.1181120	0.058627941
1.5848947	−1.0980730	0.063870189
2.5118893	−1.0754678	0.066832765
3.9810768	−1.0512315	0.065599039
6.3095828	−1.0272895	0.060118625
10.000016	−1.0063910	0.048717882
15.848960	−0.99185120	0.032878953
25.118912	−0.98612750	0.016017054

TABLE I — *Concluded*

Frequency, mc	v_{12}/v_{11} (r.p.)	v_{12}/v_{11} (i.p.)
0.10000000	0.84589800	0.018268992
0.15848935	0.85192153	0.023529840
0.25118873	0.85932664	0.026878194
0.39810738	0.86810431	0.034308091
0.63095778	0.87885470	0.040451987
1.0000008	0.89191254	0.046767278
1.5848947	0.90761553	0.052792147
2.5118893	0.92625098	0.057560082
3.9810768	0.94753039	0.059488464
6.3095828	0.97011293	0.056772712
10.000016	0.99132599	0.047989710
15.848960	1.0071079	0.033385241
25.118912	1.0137993	0.016466337

Frequency, mc	v_{32}/v_{31} (r.p.)	v_{32}/v_{31} (i.p.)
0.10000000	−2.9917880	2.7097173
0.15848935	−2.1786147	2.2242917
0.25118873	−1.6316601	1.7969975
0.39810738	−1.2145871	1.4571936
0.63095778	−0.88363188	1.1750175
1.0000008	−0.62260636	0.94088943
1.5848947	−0.41596602	0.74652410
2.5118893	−0.25394751	0.58298677
3.9810768	−0.13047707	0.44414391
6.3095828	−0.040132660	0.32580464
10.000016	0.021255900	0.22404223
15.848960	0.055588468	0.13656873
25.118912	0.063935269	0.064322522

Frequency, mc	v_{11}/i_{11} (r.p.), ohms	v_{11}/i_{11} (i.p.), ohms
0.10000000	−77.329377	−1.1101280
0.15848935	−78.639675	−6.2732360
0.25118873	−80.548062	−11.518517
0.39810738	−83.030268	−17.666091
0.63095778	−86.205314	−25.523024
1.0000008	−89.947652	−36.084481
1.5848947	−93.776202	−50.973284
2.5118893	−96.541720	−72.748214
3.9810768	−95.405167	−105.27411
6.3095828	−83.247510	−154.12323
10.000016	−42.817517	−225.93510
15.848960	66.135311	−321.23818
25.118912	332.08865	−400.61598

Frequency, mc	v_{31}/i_{31} (r.p.), ohms	v_{31}/i_{31} (i.p.), ohms
0.10000000	−5.7971713	0.30779950
0.15848935	−5.9324594	−0.10551400
0.25118873	−6.0322898	−0.46269910
0.39810738	−6.1941820	−0.89163540
0.63095778	−6.5156971	−1.3739212
1.0000008	−7.0303966	−1.8609257
1.5848947	−7.7530091	−2.2944494
2.5118893	−8.6694443	−2.5937969
3.9810768	−9.7168424	−2.6696293
6.3095828	−10.780327	−2.4534229
10.000016	−11.699977	−1.9224920
15.848960	−12.286170	−1.1462743
25.118912	−12.411509	−0.33026606

TABLE II — TRANSPOSED THREE-CONDUCTOR COAXIAL LINE

$$\gamma_{Ti} = \alpha_{Ti} + j\beta_{Ti}$$

Transposition Interval $2l$ = 73 ft 9 in.

Frequency, mc	$10^2\alpha_{Ti}2l$ nepers	$\beta_{Ti}2l$ radians	Total Attenuation, db/1180 ft.
0.1000	1.204	0.07565	1.674
0.1585	1.257	0.1187	1.747
0.2512	1.320	0.1871	1.834
0.3981	1.399	0.2955	1.944
0.6310	1.519	0.4679	2.111
1.0000	1.776	0.7471	2.468
1.585	4.018	1.209	5.584
1.995	15.56	1.475	21.63
2.512	9.072	1.415	12.61
3.162	4.387	0.9181	6.097
3.981	3.831	0.3169	5.325
5.012	4.253	0.4368	5.991
6.310	14.52	1.367	20.17
7.943	5.346	0.6606	7.429
10.00	7.610	0.8457	10.58
12.60	7.044	0.5103	9.789
15.85	10.51	1.0378	14.61

Transposition Interval $2l$ = 4 ft $7\frac{5}{8}$ in.

Frequency, mc	$10^3\alpha_{Ti}2l$ nepers	$\beta_{Ti}2l$ radian	Total Attenuation, db/1057 ft.
1.000	1.214	0.04610	2.404
1.585	1.322	0.07292	2.618
2.512	1.545	0.1153	3.059
3.981	1.867	0.1825	3.698
6.310	2.543	0.2890	5.035
10.00	3.770	0.4579	7.467
12.60	4.767	0.5767	9.440
15.85	6.185	0.7273	12.25

Transposition Interval $2l$ = 9 ft $3\frac{1}{4}$ in.

Frequency, mc	$10^3\alpha_{Ti}2l$ nepers	$\beta_{Ti}2l$ radians	Total Attenuation, db/1057 ft.
0.1000	1.506	0.009458	1.491
0.1585	1.574	0.01486	1.558
0.2512	1.655	0.02337	1.638
0.3981	1.756	0.03690	1.739
0.6310	1.900	0.05830	1.882
1.000	2.124	0.09221	2.103
1.585	2.464	0.1459	2.440
2.512	2.990	0.2308	2.961
3.981	3.804	0.3655	3.767
6.310	5.301	0.5798	5.249
10.00	9.102	0.9258	9.012
12.60	16.45	1.185	16.29

Table II — *Concluded*

Frequency, mc	$10^2\alpha_{T3}2l$ nepers	$\beta_{T3}2l$ radians	
0.1000	1.2450	0.01740	
0.1585	1.4960	0.02391	
0.2512	1.776	0.03354	
0.3981	2.072	0.04826	
0.6310	2.381	0.07121	
1.000	2.715	0.1073	
1.585	3.092	0.1640	
2.512	3.546	0.2535	
3.981	4.126	0.3947	
6.310	4.932	0.6180	
10.00	6.214	0.9763	
12.60	7.533	1.243	

$$\gamma_{T1} = \alpha_{T1} + j\beta_{T1}$$

Transposition Interval — Infinitesimal

5-mil Dielectric

Frequency, mc	$10^3\alpha_{T1}$ nepers/meter	β_{T1} radian/meter	Total Attenuation, db/1057 ft.
0.1000	0.5329	0.003347	1.419
0.1585	0.5569	0.005521	1.558
0.2512	0.5856	0.008271	1.638
0.3981	0.6215	0.01306	1.739
0.6310	0.6725	0.02063	1.882
1.000	0.7504	0.03263	2.100
1.585	0.8716	0.05161	2.439
2.512	1.053	0.08163	2.947
3.981	1.326	0.1291	3.711
6.310	1.780	0.2043	4.980
10.00	2.586	0.3233	7.234
15.85	3.944	0.5113	11.04
19.95	4.842	0.6430	13.55

1-mil Dielectric

Frequency, mc	$10^3\alpha_{T1}$ nepers/meter	β_{T1} radian/meter	Total Attenuation, db/1057 ft.
0.1000	0.5109	0.003331	1.429
0.1585	0.5330	0.005232	1.491
0.2512	0.5565	0.008248	1.557
0.3981	0.5867	0.01303	1.642
0.6310	0.6308	0.02060	1.765
1.000	0.6999	0.03258	1.958
1.585	0.8098	0.05155	2.266
2.512	0.9770	0.08156	2.734
3.981	1.232	0.1290	3.447
6.310	1.663	0.2042	4.652
10.00	2.439	0.3232	6.825
15.85	3.762	0.5112	10.53
19.95	4.368	0.6428	12.98

Table III — Two-Conductor Line — $\gamma = \alpha + j\beta$

Frequency, mc	$10^3\alpha$ nepers/meter	β radian/meter	Total Attenuation, db/1057 ft.
0.1000	0.3350	0.003342	0.9373
0.2000	0.4252	0.006549	1.190
0.4000	0.5547	0.01292	1.552
0.7000	0.7055	0.02242	1.974
1.000	0.8344	0.03188	2.335
2.000	1.186	0.06332	3.320
4.000	1.687	0.1259	4.721
7.000	2.227	0.2197	6.232
10.00	2.659	0.3133	7.441
14.00	3.145	0.4380	8.799
20.00	3.756	0.6250	10.51
25.00	4.198	0.7808	11.75

In the case of infinitesimal spacing of transpositions, the propagation constant γ_{T1} for the low-loss mode was computed from (35) for both the actual cable and a hypothetical one in which the intermediate conductor was separated from the inner conductor by only 1 mil instead of 5 mils. These quantities appear at the end of Table II.

In order to have a calculated curve to compare these two results with, the propagation constant of the two-conductor reference cable was also computed by modifying the process used for the uniform line. These quantities are shown in Table III.

The numbers delivered by the computer were rounded off to four significant digits in Tables II and III. In the table of results for the 9 ft $3\frac{1}{4}$ in. interval, the values for infinitesimal spacing were used up to 1 mc because the eight-digit precision of the computer was not sufficient to calculate the roots of (32) at these frequencies. At 1 mc the two methods give answers differing by about 0.1 per cent.

VII. ACKNOWLEDGMENT

The project described in this paper involved the efforts of so many people over such a long period of time that individual mention of their contributions is impossible. Certain individuals have been named at appropriate places in the text. In addition, we owe a debt to W. H. Doherty, R. K. Potter and R. L. Wallace, who stimulated this project in its early stages; to C. V. LoBianco, who made many contributions to the fabrication, measurement and computations; to P. H. Smith and J. M. Olsen; and to the members and former members of the Transmission Research, Outside Plant Development and Mathematical Research Departments of Bell Telephone Laboratories, without whose cooperation

and assistance this project could not have been carried to a successful conclusion.

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